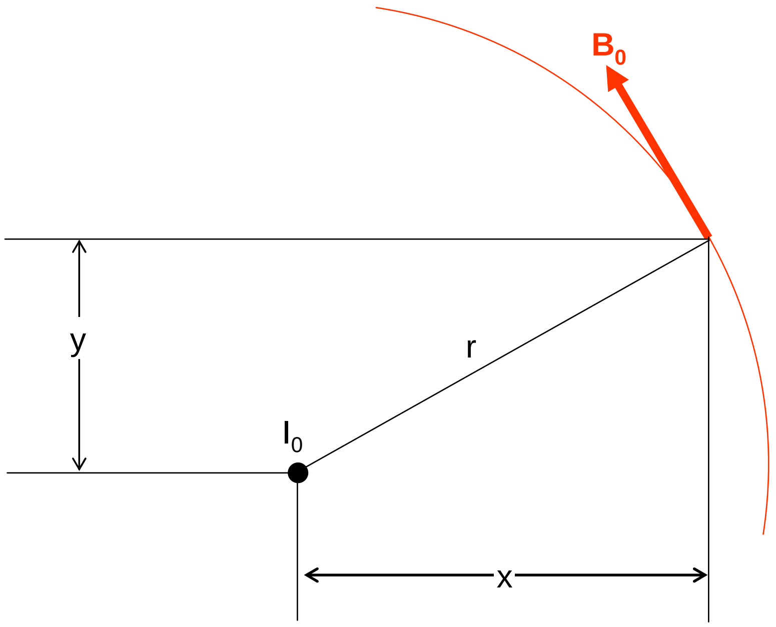
How to calculate the magnetic field from a three-phase circuit

We will consider here a single three-phase circuit with infinitely long, straight conductors.

To build up the calculation in stages, consider first just a single current, I_0.



The magnetic field it produces forms concentric circles. So at a point (x,y), the magnetic field is in the direction shown as B_0 (we have assumed the current is coming out of the page).

The size of the magnetic field is given by Ampere's law:

$$B = \frac{\mu_0 I_0}{2\pi r}$$

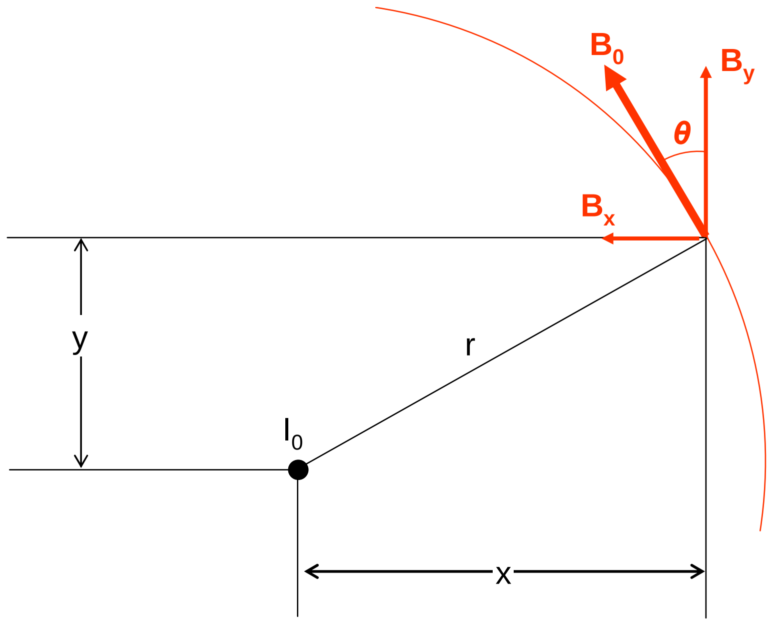
We can calculate r from Pythagoras:

$$r = \left(x^2 + y^2\right)^{\frac{1}{2}}$$

So

$$B = \frac{\mu_0 I_0}{2\pi \left(x^2 + y^2\right)^{\frac{1}{2}}}$$

Next we want to know the vertical and horizontal components of B:



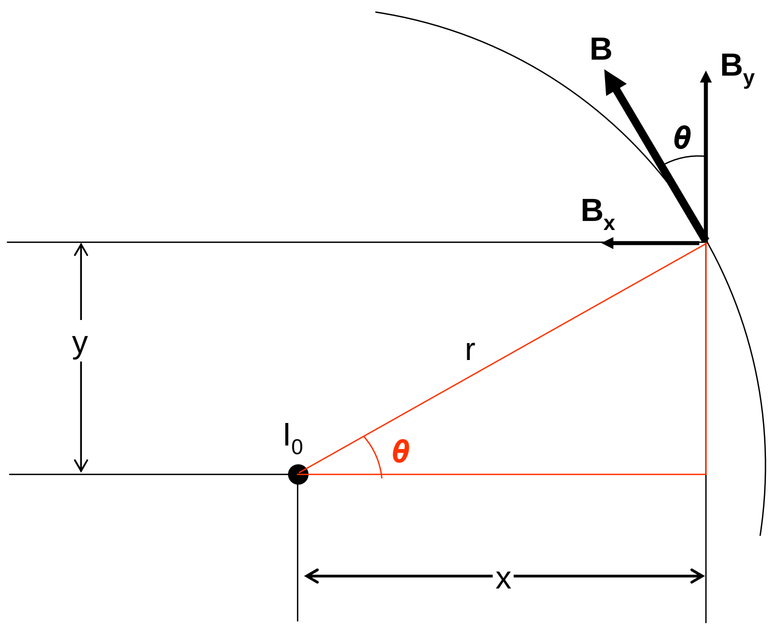
Fairly obviously

$$B_x = -B_0 \sin(\theta)$$

$$B_y = B_0 \cos(\theta)$$

(Note: with the coordinates set up the way we have, and the current assumed to be positive coming out of the page, a minus sign appears in the B_x term. With different coordinate conventions, the minus sign might be different, but the final answer would not change as long as we applied our chosen convention consistently.)

But we can calculate sin and cos of θ:



$$\cos(\theta) = \frac{x}{r} = \frac{x}{\left(x^2 + y^2\right)^{1/2}}$$

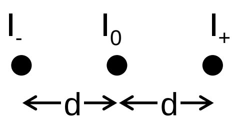
$$\sin(\theta) = \frac{y}{r} = \frac{y}{\left(x^2 + y^2\right)^{1/2}}$$

Hence

$$B_x = -\frac{\mu_0 I_0}{2\pi\left(x^2 + y^2\right)^{1/2}} \sin(\theta) = -\frac{\mu_0 I_0}{2\pi\left(x^2 + y^2\right)^{1/2}} \frac{y}{\left(x^2 + y^2\right)^{1/2}} = -\frac{\mu_0 I_0 y}{2\pi\left(x^2 + y^2\right)}$$

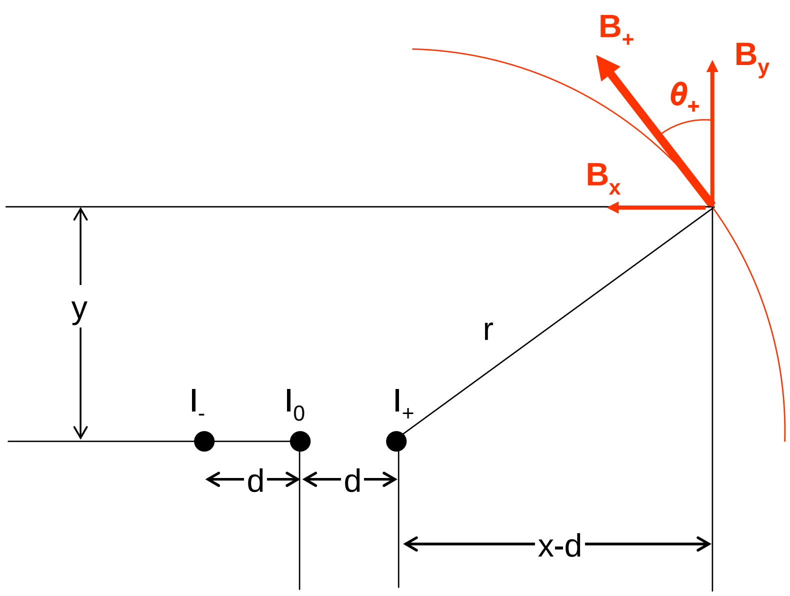
$$B_y = \frac{\mu_0 I_0}{2\pi\left(x^2 + y^2\right)^{1/2}} \cos(\theta) = \frac{\mu_0 I_0}{2\pi\left(x^2 + y^2\right)^{1/2}} \frac{x}{\left(x^2 + y^2\right)^{1/2}} = \frac{\mu_0 I_0 x}{2\pi\left(x^2 + y^2\right)}$$

Now we have calculated the field from a single current, we need to add in the other two currents to make a three-phase circuit. We choose to consider a flat circuit with conductors spaced d apart in the x direction here, but the principles apply to any geometry.



For the moment, we'll just call the three currents I_-, I_0 and I_+ - we'll worry about the actual values later.

Now consider, say, the I_+ current and the field it produces:



The geometry is just the same as for the I_0 current but with x replaced by x-d. So we can calculate the field components in exactly the same way, replacing x with x-d:

$$B_x = -\frac{\mu_0 I_+ y}{2\pi\left((x-d)^2 + y^2\right)}$$

$$B_y = \frac{\mu_0 I_+ (x-d)}{2\pi\left((x-d)^2 + y^2\right)}$$

The I_- terms are just the same but with x+d. (If we had a vertical array instead of a horizontal array, the "d" terms would appear with the y terms instead of the x, and if the array was more complicated, there would be modifications to both the x and y terms.) So now we have the geometrical factors for the vertical and horizontal components for all three currents:

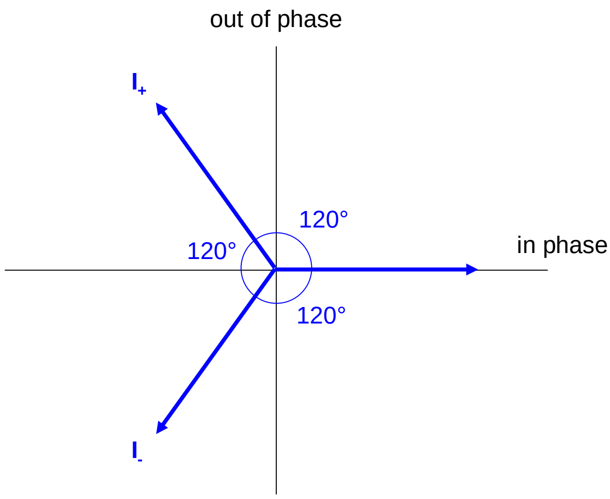
	I_-	I_0	I_+
B_x	$-\frac{\mu_0 y}{2\pi\left((x+d)^2 + y^2\right)}$	$-\frac{\mu_0 y}{2\pi\left(x^2 + y^2\right)}$	$-\frac{\mu_0 y}{2\pi\left((x-d)^2 + y^2\right)}$
B_y	$\frac{\mu_0 (x+d)}{2\pi\left((x+d)^2 + y^2\right)}$	$\frac{\mu_0 x}{2\pi\left(x^2 + y^2\right)}$	$\frac{\mu_0 (x-d)}{2\pi\left((x-d)^2 + y^2\right)}$

Now we need to think about the relation of the three currents to each other. In a single circuit, the three currents will be given by:

I_-	I_0	I_+
$I\sin(\omega t - 120°)$	$I\sin(\omega t)$	$I\sin(\omega t + 120°)$

With just one circuit, it actually really doesn't matter which order we put these currents in, each individual current will always have a phase 120° apart from the other two. If we have more than one circuit, then we would need a consistent convention over how we described the phases.

We now need to resolve each current into an in-phase and an out-of-phase component. We do this using a phasor diagram:



Using this, and the sin and cos of 30°, we get:

	I_-	I_0	I_+
I_{in}	$-\frac{1}{2}I$	I	$-\frac{1}{2}I$
I_{out}	$-\frac{\sqrt{3}}{2}I$	0	$\frac{\sqrt{3}}{2}I$

So now we can work out, for each current, the in-phase vertical, out-of-phase vertical, in-phase horizontal, and out-of-phase horizontal components, by multiplying the geometrical term by the appropriate current term. To work this through for just one component, let's consider the out-of-phase vertical field for I_+:

Geometrical factor: $\frac{\mu_0(x-d)}{2\pi\left((x-d)^2+y^2\right)}$

Current term: $\frac{\sqrt{3}}{2}I$

Resulting component of the field: $\frac{\mu_0(x-d)}{2\pi\left((x-d)^2+y^2\right)}\frac{\sqrt{3}}{2}I$

Now, sticking with this example, there are three out-of-phase vertical components (although the middle one happens to be zero), one from each current, and they just add together:

$$B_{out,y}=\frac{\mu_0(x+d)}{2\pi\left((x+d)^2+y^2\right)}\frac{-\sqrt{3}}{2}I+\frac{\mu_0 x}{2\pi\left(x^2+y^2\right)}0I+\frac{\mu_0(x-d)}{2\pi\left((x-d)^2+y^2\right)}\frac{\sqrt{3}}{2}I$$

Clearly, these terms would simplify, but usually you would be doing the calculations numerically so there's no great advantage in writing out the simplified terms.

If we follow this through, we get four components of field:

$$B_{in,x} \quad B_{in,y} \quad B_{out,x} \quad B_{out,y}$$

where each of these in turn is the sum of three components, one from each of the currents. To get the overall total, we add these as root-sum-of-squares:

$$B_{total}=\sqrt{B_{in,x}^2+B_{in,y}^2+B_{out,x}^2+B_{out,y}^2}$$

Note that we defined *I* as the amplitude of the current. If we use the amplitude of the current, we get the amplitude of the field. But equally, if we use the rms current, we get the rms field, and this is what we would usually do.

More complicated situations:

- If you have more than one circuit, work out all the separate components, sum the in-phase and out-of-phase, horizontal and vertical components from every separate conductor, then take the root-sum-of-squares total as before.
- If the currents aren't exactly at 120° to each other, use the actual angles and work out the sin and cos terms from the phasor diagram numerically.

- If the currents aren't infinitely long or straight, break them into short straight sections and calculate the components for each of these separately (you can no longer use Ampere's law as it stands, obviously, you have to reduce it by the sin of the angle subtended by the ends of the section at the point of interest).